

ALGEBRAIC NUMBER THEORY- SHEET 8

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Solutions to 8.3 and 8.4 should be handed in to me via Moodle by **11:59PM on 28/03/2021**.

Exercise 8.1. (1) Let $K = \mathbb{Q}(\sqrt{-13})$. Compute the class group Cl_K and give the multiplication table.

(2) Find all integer solutions to the Diophantine equation $x^3 = y^2 + 13$, justifying your answers carefully.

Exercise 8.2. (1) Let $K = \mathbb{Q}(\sqrt{-17})$. Compute the class group Cl_K and give the multiplication table.

(2) Find all integer solutions to the Diophantine equation $x^3 = y^2 + 17$, justifying your answers carefully.

Exercise 8.3. (1) Let $K = \mathbb{Q}(\sqrt{-79})$. Compute the class group Cl_K and give the multiplication table.

(2) Find all integer solutions to the Diophantine equation $x^3 = y^2 - y + 20$, justifying your answers carefully.

Exercise 8.4. Let ζ_7 be a 7-th root of unity and $K = \mathbb{Q}(\zeta_7)$. Complete the following table describing the decomposition of ideals (p) (with p a prime number) in \mathcal{O}_K as done in Example 3.6.16 of the notes. In the table, we let $n = p^k m$ where $p \nmid m$.

$p \bmod 7$	Order of $p \bmod m$	Factorization of (p)	Norms
0			
1			
2			
3			
4			
5			
6			

Exercise 8.5. Please procrastinate by going to moodle and doing the student evaluation questionnaire.