

and flag is fried as its Essenstein with 23

Remark 2.2.11. Note that this also gives us an algorithm for finding an integral basis as follows:

- 1. Pick B a basis consisting of algebraic integers and calculate $\Delta(B)$.
- 2. For each prime p such that $p^2 \mid \Delta(B)$ we can use Lemma 2.2.7 to get a new basis B' with smaller discriminant.
- 3. Now, repeat step one.

Step 2 Take B= ?1,d,d,d, 25} os our initial boss Compute 1(B)

> **Theorem 2.2.25.** Let $K = \mathbb{Q}(\alpha)$ a number field with $m_{\alpha}(x) = x^n + ax + b$. Then $\Delta(\{1,\alpha,\ldots,\alpha^{n-1}\}) = (-1)^{\frac{n(n-1)}{2}} (n^n b^{n-1} + (-1)^{n-1} (n-1)^{n-1} a^n).$

$$\Delta(3) = (-1)^{6}(4^{4}.(-3)^{3}) = -2^{8}.3^{3}$$

ve flore: 1.8, -. 3 Note

Lemma 2.2.7. Let K be a number field and $B = \{b_1, \ldots, b_n\}$ be a basis for K/\mathbb{Q} consisting of algebraic integers. If B is not an integral basis then there exists an algebraic integer of the form

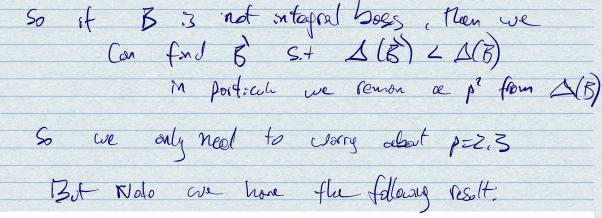
$$\alpha = \frac{x_1b_1 + \dots + x_nb_n}{p}$$

$$\sum_{k=1}^{\infty} \frac{\sum_{k=1}^{\infty} \sum_{k=1}^{\infty} \sum_{k=1$$

where p is a prime and $x_i \in \{0, ..., p-1\}$ with not all x_i zero. Moreover, if $x_i \neq 0$ and we let B' be the basis obtained by replacing b_i with α , then

$$\Delta(B') = \frac{x_i^2}{p^2} \Delta(B).$$

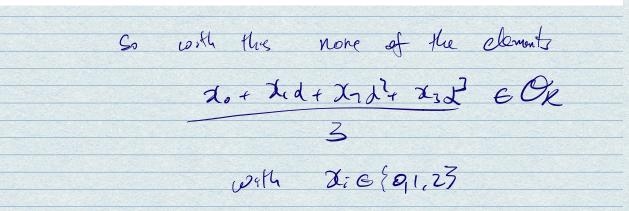
In particular $p^2 \mid \Delta(B)$.



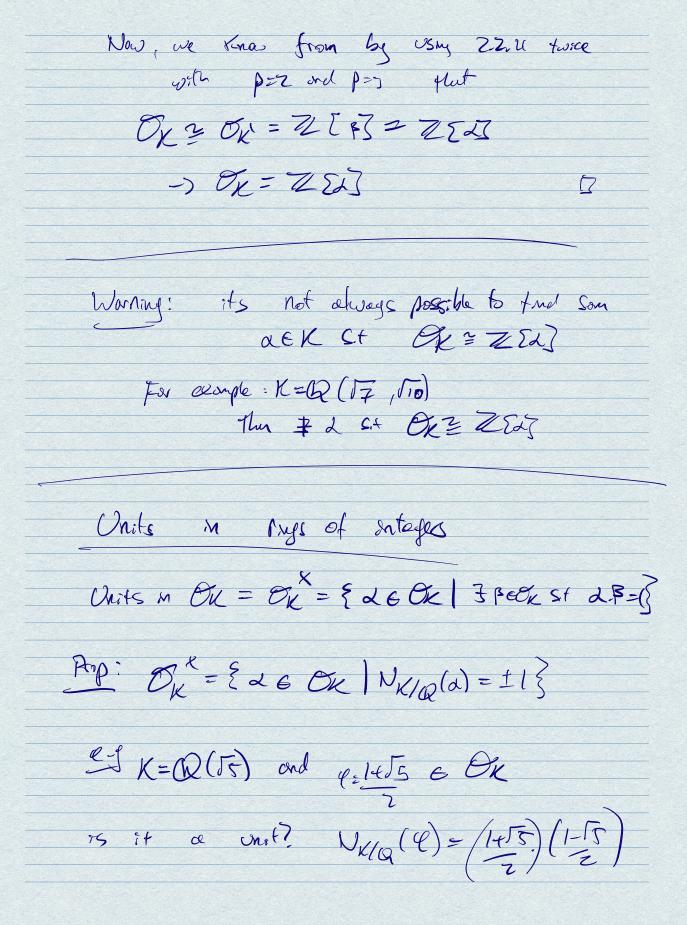
Lemma 2.2.21. Let $K = \mathbb{Q}(\alpha)$ and α be an algebraic integer such that m_{α} satisfies Eisensteins Criterion 1.2.15 for a prime p. Then none of the elements

$$\phi = \frac{1}{p}(x_0 + x_1\alpha + \dots + x_{n-1}\alpha^{n-1})$$

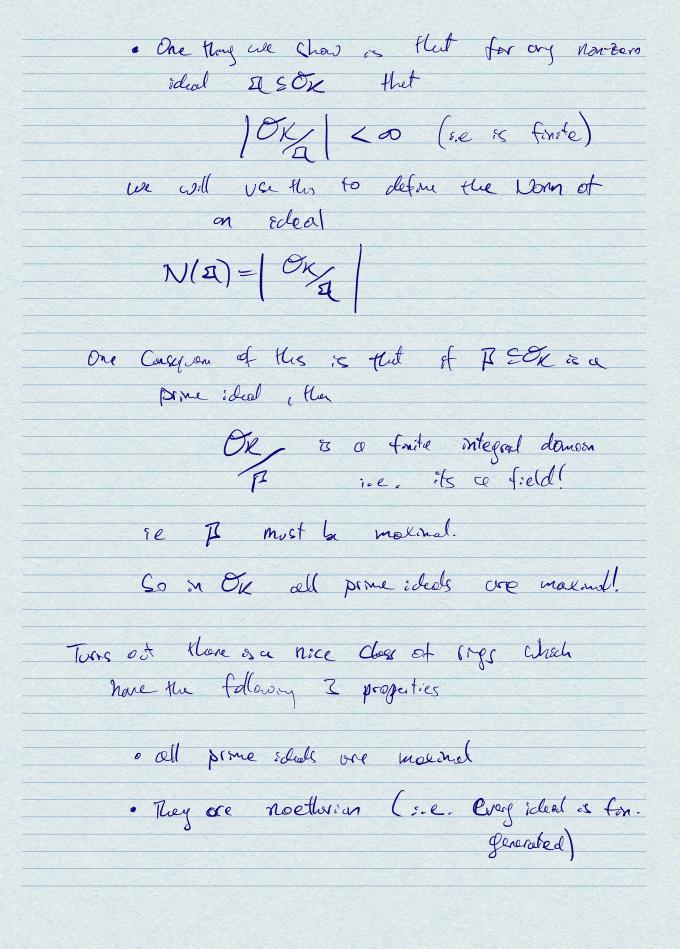
is an algebraic integer, where $n = \deg(m_{\alpha})$ and $x_i \in \{0, \dots, p-1\}$.



So we don't have to warry about PEZ left with p=2. i.e we need to check if ony demont of the form Lot Ned + Ardit Xsd & OK with die {0,17. The con be done Set flure is a botter croy: Hirt! Ford a Charge of Vorsables to make 24-3 Essuten of P=2 and 3. Note that f(x) = (x+3)9-3 = 24 + 12x1 + 5422+108x+78 This is Essueta of p=2 and 3. let & be a root of f(X) == d+3 K=Q(5) then we know that A({1, B, , B3}) = M(1, 2, -, 23) By Q. S. 6 of the pods Sheet KIRK=Q(d) -> OK ZOK



= 1-5=-1
Theorem (Dirschlets Onst theorem)
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