

Here is a table for the 65 values of n we can find a congruence condition on primes that will guarantee we write the prime as $x^2 + ny^2$.

n	Coungrence classes for primes
1	$p \equiv 1 \pmod{4}$
2	$p \equiv 1, 3 \pmod{8}$
3	$p = 3$ or $p \equiv 1 \pmod{3}$
4	$p \equiv 1, 5, 9, 13 \pmod{16}$
5	$p \equiv 1, 9 \pmod{20}$
6	$p \equiv 1, 7 \pmod{24}$
7	$p = 7$ or $p \equiv 1, 9, 11, 15, 23, 25 \pmod{28}$
8	$p \equiv 1, 9, 17, 25 \pmod{32}$
9	$p \equiv 1, 13, 25 \pmod{36}$
10	$p \equiv 1, 9, 11, 19 \pmod{40}$
12	$p \equiv 1, 13, 25, 37 \pmod{48}$
13	$p = 13$ or $p \equiv 1, 9, 17, 25, 29, 49 \pmod{52}$
15	$p \equiv 1, 19, 31, 49 \pmod{60}$
16	$p \equiv 1, 9, 17, 25, 33, 41, 49, 57 \pmod{64}$
18	$p \equiv 1, 19, 25, 43, 49, 67 \pmod{72}$
21	$p \equiv 1, 25, 37 \pmod{84}$
22	$p \equiv 1, 9, 15, 23, 25, 31, 47, 49, 71, 81 \pmod{88}$
24	$p \equiv 1, 25, 49, 73 \pmod{96}$
25	$p \equiv 1, 9, 21, 29, 41, 49, 61, 69, 81, 89 \pmod{100}$
28	$p \equiv 1, 9, 25, 29, 37, 53, 57, 65, 81, 85, 93, 109 \pmod{112}$
30	$p \equiv 1, 31, 49, 79 \pmod{120}$
33	$p \equiv 1, 25, 37, 49, 97 \pmod{132}$
37	$p = 37$ or $p \equiv 1, 9, 21, 25, 33, 41, 49, 53, 65, 73, 77, 81, 85, 101, 121, 137, 141, 145 \pmod{148}$
40	$p \equiv 1, 9, 41, 49, 81, 89, 121, 129 \pmod{160}$
42	$p \equiv 1, 25, 43, 67, 121, 163 \pmod{168}$
45	$p \equiv 1, 49, 61, 109, 121, 169 \pmod{180}$
48	$p \equiv 1, 25, 49, 73, 97, 121, 145, 169 \pmod{192}$
57	$p \equiv 1, 25, 49, 61, 73, 85, 121, 157, 169 \pmod{228}$
58	$p \equiv 1, 9, 25, 33, 35, 49, 51, 57, 59, 65, 67, 81, 83, 91, 107, 115, 121, 123, 129, 139, 161, 169, 179, 187, 209, 219, 225, 227 \pmod{232}$
60	$p \equiv 1, 49, 61, 109, 121, 169, 181, 229 \pmod{240}$
70	$p \equiv 1, 9, 39, 71, 79, 81, 121, 151, 169, 191, 239, 249 \pmod{280}$
72	$p \equiv 1, 25, 49, 73, 97, 121, 145, 169, 193, 217, 241, 265 \pmod{288}$
78	$p \equiv 1, 25, 49, 55, 79, 103, 121, 127, 199, 217, 289, 295 \pmod{312}$
85	$p \equiv 1, 9, 21, 49, 69, 81, 89, 101, 121, 149, 161, 169, 189, 229, 281, 321 \pmod{340}$
88	$p \equiv 1, 9, 17, 25, 41, 49, 57, 65, 73, 81, 89, 97, 105, 113, 129, 137, 145, 153, 161, 169, 177, 185, 193, 201, 217, 225, 233, 241, 249, 257, 265, 273, 281, 289, 305, 313, 321, 329, 337, 345 \pmod{352}$
93	$p \equiv 1, 25, 49, 97, 109, 121, 133, 157, 169, 193, 205, 253, 289, 349, 361 \pmod{372}$
102	$p \equiv 1, 25, 49, 55, 103, 121, 127, 145, 151, 169, 217, 223, 247, 271, 319, 361 \pmod{408}$
105	$p \equiv 1, 109, 121, 169, 289, 361 \pmod{420}$

112	$p \equiv 1, 9, 25, 57, 65, 81, 113, 121, 137, 169, 177, 193, 225, 233, 249, 281, 289, 305, 337, 345, 361, 393, 401, 417 \pmod{448}$
120	$p \equiv 1, 49, 121, 169, 241, 289, 361, 4091 \pmod{480}$
130	$p \equiv 1, 9, 49, 51, 81, 121, 129, 131, 139, 179, 209, 211, 251, 259, 289, 321, 329, 339, 361, 419, 441, 451, 459, 491 \pmod{520}$
133	$p \equiv 1, 9, 25, 81, 85, 93, 121, 137, 149, 169, 177, 197, 225, 233, 253, 277, 289, 305, 309, 365, 389, 429, 457, 473, 501, 505, 529 \pmod{532}$
165	$p \equiv 1, 49, 169, 181, 229, 289, 301, 361, 421, 529 \pmod{660}$
168	$p \equiv 1, 25, 121, 169, 193, 289, 337, 361, 457, 505, 529, 625 \pmod{672}$
177	$p \equiv 1, 25, 49, 85, 121, 133, 145, 169, 181, 193, 205, 241, 253, 265, 277, 289, 361, 373, 433, 481, 493, 517, 529, 553, 577, 625, 661, 685, 697 \pmod{708}$
190	$p \equiv 1, 9, 39, 49, 81, 111, 119, 121, 159, 161, 169, 191, 199, 201, 239, 271, 289, 311, 321, 329, 351, 359, 391, 441, 479, 481, 511, 519, 529, 609, 631, 671, 681, 689, 719, 729 \pmod{760}$
210	$p \equiv 1, 121, 169, 211, 289, 331, 361, 379, 499, 529, 571, 739 \pmod{840}$
232	$p \equiv 1, 9, 25, 33, 49, 57, 65, 81, 121, 129, 161, 169, 209, 225, 233, 241, 257, 265, 281, 289, 297, 313, 353, 361, 393, 401, 441, 457, 465, 473, 489, 497, 513, 521, 529, 545, 585, 593, 625, 633, 673, 689, 697, 705, 721, 729, 745, 753, 761, 777, 817, 825, 857, 865, 905, 921 \pmod{928}$
240	$p \equiv 1, 49, 121, 169, 241, 289, 361, 409, 481, 529, 601, 649, 721, 769, 841, 889 \pmod{960}$
253	$p \equiv 1, 9, 25, 49, 81, 93, 133, 141, 169, 177, 185, 213, 225, 257, 265, 269, 289, 301, 317, 353, 357, 361, 377, 397, 441, 445, 449, 485, 489, 509, 533, 537, 553, 565, 577, 581, 625, 653, 669, 685, 729, 749, 785, 817, 829, 837, 841, 905, 929, 933, 949, 961, 969, 993, 1005 \pmod{1012}$
280	$p \equiv 1, 9, 81, 121, 169, 249, 281, 289, 361, 401, 449, 529, 561, 569, 641, 681, 729, 809, 841, 849, 921, 961, 1009, 1089 \pmod{1120}$
312	$p \equiv 1, 25, 49, 121, 217, 289, 313, 337, 361, 433, 529, 601, 625, 649, 673, 745, 841, 913, 937, 961, 985, 1057, 1153, 1225 \pmod{1248}$
330	$p \equiv 1, 49, 91, 169, 289, 331, 361, 379, 499, 529, 619, 691, 841, 859, 889, 961, 1081, 1171, 1219, 1291 \pmod{1320}$
345	$p \equiv 1, 49, 121, 169, 289, 301, 349, 361, 409, 469, 541, 601, 721, 829, 841, 901, 949, 961, 1021, 1129, 1189, 1369 \pmod{1380}$
357	$p \equiv 1, 25, 121, 169, 205, 253, 361, 373, 421, 457, 529, 613, 625, 757, 781, 841, 865, 961, 1033, 1045, 1177, 1345, 1369, 1381 \pmod{1428}$
385	$p \equiv 1, 9, 81, 141, 169, 221, 289, 309, 361, 389, 401, 421, 449, 529, 641, 669, 709, 729, 841, 949, 961, 1061, 1101, 1149, 1241, 1269, 1369, 1401, 1409, 1521 \pmod{1540}$
408	$p \equiv 1, 25, 49, 121, 145, 169, 217, 361, 409, 433, 457, 529, 553, 577, 625, 769, 817, 841, 865, 937, 961, 985, 1033, 1177, 1225, 1249, 1273, 1345, 1369, 1393, 1441, 1585 \pmod{1632}$
462	$p \equiv 1, 25, 169, 247, 289, 295, 361, 463, 487, 529, 625, 631, 697, 751, 793, 823, 841, 961, 991, 1087, 1159, 1255, 1303, 1345, 1369, 1423, 1633, 1681, 1807, 1831 \pmod{1848}$
520	$p \equiv 1, 9, 49, 81, 121, 129, 209, 289, 321, 329, 361, 441, 521, 529, 569, 601, 641, 649, 729, 809, 841, 849, 881, 961, 1041, 1049, 1089, 1121, 1161, 1169, 1249, 1329, 1361, 1369, 1401, 1481, 1561, 1569, 1609, 1641, 1681, 1689, 1769, 1849, 1881, 1889, 1921, 2001 \pmod{2080}$
760	$p \equiv 1, 9, 49, 81, 121, 161, 169, 201, 289, 321, 329, 441, 481, 529, 609, 681, 689, 729, 761, 769, 809, 841, 881, 921, 929, 961, 1049, 1081, 1089, 1201, 1241, 1289, 1369, 1441, 1449, 1489, 1521, 1529, 1569, 1601, 1641, 1681, 1689, 1721, 1809, 1841, 1849, 1961, 2001, 2049, 2129, 2201, 2209, 2249, 2281, 2289, 2329, 2361, 2401, 2441, 2449, 2481, 2569, 2601, 2609, 2721, 2761, 2809, 2889, 2961, 2969, 3009 \pmod{3040}$
840	$p \equiv 1, 121, 169, 289, 361, 529, 841, 961, 1009, 1129, 1201, 1369, 1681, 1801, 1849, 1969, 2041, 2209, 2521, 2641, 2689, 2809, 2881, 3049 \pmod{3360}$

1320	$p \equiv 1, 49, 169, 289, 361, 529, 841, 889, 961, 1081, 1321, 1369, 1489, 1609, 1681, 1849, 2161, 2209, 2281, 2401, 2641, 2689, 2809, 2929, 3001, 3169, 3481, 3529, 3601, 3721, 3961, 4009, 4129, 4249, 4321, 4489, 4801, 4849, 4921, 5041 \pmod{5280}$
1365	$p \equiv 1, 121, 289, 361, 529, 589, 781, 841, 961, 1369, 1381, 1429, 1621, 1681, 1849, 2209, 2389, 2461, 2521, 2629, 2809, 2941, 3301, 3481, 3649, 3721, 3901, 3949, 4369, 4489, 4729, 4741, 5041, 5149, 5209, 5329 \pmod{5460}$
1848	$p \equiv 1, 25, 169, 289, 361, 529, 625, 697, 793, 841, 961, 1345, 1369, 1633, 1681, 1849, 1873, 2017, 2137, 2209, 2377, 2473, 2545, 2641, 2689, 2809, 3193, 3217, 3481, 3529, 3697, 3721, 3865, 3985, 4057, 4225, 4321, 4393, 4489, 4537, 4657, 5041, 5065, 5329, 5377, 5545, 5569, 5713, 5833, 5905, 6073, 6169, 6241, 6337, 6385, 6505, 6889, 6913, 7177, 7225 \pmod{7392}$